EMH and RW

CLM Chapter 2

Data and Statistics

- It is important to understand the difficulties associated with estimating something as simple as the mean return.

- We focus on now are the properties of estimates of
  - Means
  - Other moments

Mean and Variance Estimation

- The sample mean and sample variance are given by:

$$\bar{x} = \sum_{i=1}^{T} x_i$$

$$s^2 = \frac{1}{T-1}\sum_{i=1}^{T}(x_i - \bar{x})^2 \approx \frac{1}{T}\sum_{i=1}^{T} x_i^2 - (\bar{x})^2$$

Note: For financial series with frequency lower than monthly, the second term is very small (negligible) relative to the first. It can be ignored.

Example: Weekly Japanese returns (1980-1996, T=847)
Mean = 0.257% and SD = 3.095%.
The squared average return is $(847/846)(.00257)^2 = .0000066$. The variance term is $(.03095)^2 = .0009579$, which is much greater than the squared mean return.

Sample Distributions and Estimation Error

- Sample mean is distributed $N(\mu, \sigma^2/T)$
- **Example**: C.I. for the mean of Japanese weekly stock returns again.

  Point estimate of $\mu = 0.257\%$

  Point estimate of $\sigma^2 = 3.095\%$

  Using the sample estimates: $SE(\bar{x}) = s/T^{1/2} = (.03095)/(847)^{1/2} = .00106$ (or .11%).

  95% CI is given by (.037%, .467%). It is significantly different that zero.

  But, the CI is quite wide. After 17 years, the expected return, μ, is measured with low precision.

• Sample variance is distributed $(T-1)\,s^2/\sigma^2 \sim \chi^2_{T-1}$.

• To derive a C.I. for the variance, let rewrite the standard confidence interval for a chi-squared variable:

$$P(\chi^2_{v,1-\alpha/2} < \chi^2_v < \chi^2_{v,\alpha/2}) = P(\chi^2_{v,1-\alpha/2} < (T-1)s^2/\sigma^2 < \chi^2_{v,\alpha/2}) = 1-\alpha$$

After some easy algebra, we derive:

$$P[(T-1)s^2/\chi^2_{v,\alpha/2} < \sigma^2 < (T-1)s^2/\chi^2_{v,1-\alpha/2}] = 1-\alpha.$$

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Example: C.I. for variance Japanese weekly stock returns.
We want to calculate a 95% confidence interval for σ^2 --i.e., $\alpha=.05$.
We get: $\chi^2_{846,.975}=767.3$ and $\chi^2_{846,.025}=914.8$.

$P[(846)(.03095)^2/(914.8) < \sigma^2 < (846)(.03095)^2/(767.3)] = .95$
$P[\,.000886 < \sigma^2 < .001056\,] = .95$
$\qquad$ => a 95% C.I. is given by (2.98%,3.25%).

The CI is quite compact around the sample point estimate. Compared to the sample mean estimate, σ is measured with accuracy.

Note: Usually T is larger than 30. We can use the normal approximation to calculate CIs for the population σ. For the Japanese data, we estimate the S.E. for the sample SD:
SE(s) = s/(2 T)1/2 = .03095/(2 847)1/2 = .000752 (or .075%).
A 95% CI for σ is given by (2.945%,3.245%). (Good approximation!)

Empirical Validation

- CLM and the usual empirical work present the empirical properties of stock returns.
- They are not really consistent with either the simple normal or the lognormal models.
- You need to note the differences and see how things might be improved.
- CLM's Chapter 2 considers the predictability of asset returns.
- Q: Can past return realizations tell us anything about expected future returns?
- The *efficient markets hypothesis* (EMH) is a first attempt to address the predictability issue.

General Problem

- Recall the Japanese weekly returns example. Even with 17 years of weekly information, returns are estimated with low precision.

- Estimating means with precision requires a very long T. We don't have that long history.

EMH

- Earliest known version:

   "When shares become publicly known in an open market, the value which they acquire there may be regarded as the judgement of the best intelligence concerning them."
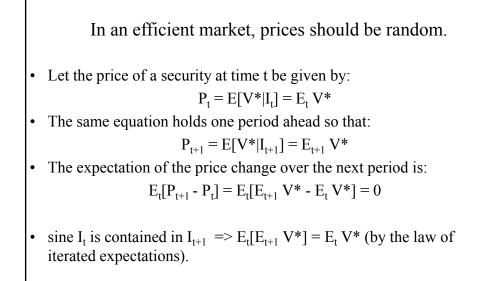
   - George Gibson, *The Stock Exchanges of London Paris and New York*, G. P. Putnman & Sons, New York, 1889

- Fama (1970)

   "A market in which prices always `fully reflect` available information is `efficient`."

- Malkiel (1992)

   "A capital market is fully efficient if it correctly reflects all information in determining security prices. Formally, the market is said to be efficient with respect to some information set... if security prices would be unaffected by revealing that information to all market participants. Moreover, efficiency with respect to an information set... implies that it is impossible to make economic profits by trading on the basis of [the information in that set]."

In an efficient market, prices should be random.

- Let the price of a security at time t be given by:
$$P_t = E[V^*|I_t] = E_t V^*$$
- The same equation holds one period ahead so that:
$$P_{t+1} = E[V^*|I_{t+1}] = E_{t+1} V^*$$
- The expectation of the price change over the next period is:
$$E_t[P_{t+1} - P_t] = E_t[E_{t+1} V^* - E_t V^*] = 0$$

- sine I_t is contained in I_{t+1} $\Rightarrow E_t[E_{t+1} V^*] = E_t V^*$ (by the law of iterated expectations).

Discussion

- The second sentence of Malkiel's definition expands Fama's definition and suggests a test for efficiency useful in a laboratory.

- The third sentence suggests a way to judge efficiency that can be used in empirical work.
  - This is what is concentrated on in the finance literature.
  - Examples: mutual fund managers profits if they are true economic profits then prices are not efficient with respect to their information.
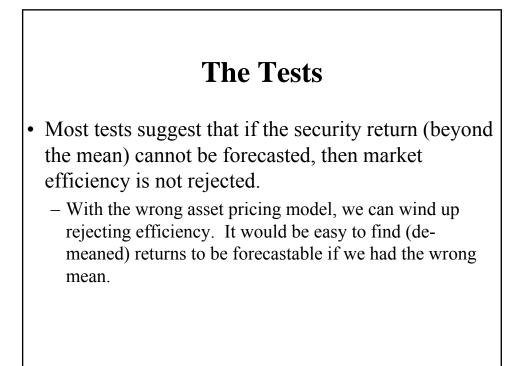 - Difficult to test for good reasons we will discuss.

Versions of Efficiency

• Efficiency can only be defined with reference to a specific type of information. Three classes of information:

> (a) Historical sequence of prices (weak form EMH).
>
> (b) Public records of companies and public forecasts regarding the future performance and possible actions (a + b: => semi-strong form EMH).
>
> (c) Private or inside information (a + b + c => strong form EMH).

Violations of Efficiency

• Technical traders devising profitable strategies (weak EMH)
• Reading a newspaper and devising a profitable trading strategy (semi-strong EMH)
• Corporate insiders making profitable trades (strong EMH)

• Q: Can markets really be strong-form efficient?

• Perfectly rational factors may account for violations of EMH:
  – Microstructure issues and Trading cost
  – Rewarding investors for bearing certain dynamic risks.
  – Time-varying expected returns due to changing conditions can generate predictability

What Does "Profitable" Mean?

- We're talking about economic profits, adjusting for risk and costs.
  - We need a model for risk adjustment. Results will be conditional on the model.

- That is, tests of efficiency are *joint tests* of efficiency and some asset pricing model, or benchmark.
  - For example, many benchmarks typically assume constant "normal" returns. This is easier to implement, but doesn't have to be right. Hence rejections of efficiency could be due to rejections of the benchmark.

The Tests

- Most tests suggest that if the security return (beyond the mean) cannot be forecasted, then market efficiency is not rejected.
  - With the wrong asset pricing model, we can wind up rejecting efficiency. It would be easy to find (de-meaned) returns to be forecastable if we had the wrong mean.

Martingales and Fair Games

- **Martingale:** A stochastic process PX_t is a martingale if:

    $$E[P_{t+1} \mid \Omega_t] = P_t \qquad \text{or } E_t[P_{t+1}] = P_t$$

    Submartingale if $E[P_{t+1} \mid \Omega_t] \geq P_t$, and supermartingale if $E[P_{t+1} \mid \Omega_t] \leq P_t$

- **Fair game model**: A stochastic process r_t is a fair game if:

    $$E[r_{t+1} \mid \Omega_t] = 0$$

    $\Rightarrow$ if P_t is a martingale or pure random walk, $(P_{t+1} - P_t)$ is a fair game

    <u>Note</u>: Only referring to expected values! Nothing about variances of shocks.

• The Martingale process is an AR(1) process with the AR coefficient equal to 1. A non-stationary process.

• LeRoy (1973), Lucas (1978): The Martingale condition is neither a necessary nor a sufficient condition for rational expectations models of asset prices.

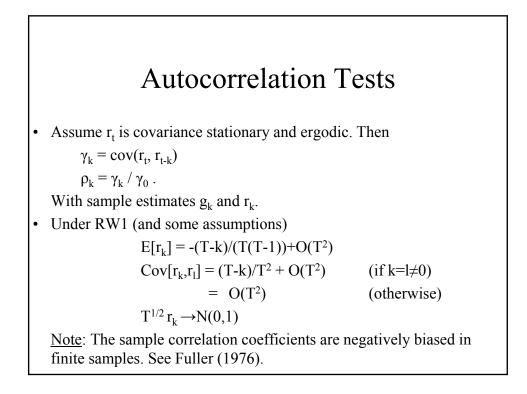• But, we can consider the martingale as an important starting point.

The Random Walk Hypotheses

- Random Walk (RW): A stochastic process p_t is a RW if:

$$p_t = \mu + p_{t-1} + \varepsilon_t \qquad \text{-where } p_t = \ln(P_t)$$

$$\varepsilon_{t+1} \sim D(0, \sigma^2), \ E_t[\varepsilon_{t+1}] = 0, \ E_t[\varepsilon^2_{t+1}] = \sigma^2$$

$$\Rightarrow r_t = \mu + \varepsilon_t = \Delta p_t$$

<u>Note</u>: A RW is a martingale with constant variance of the innovations.

- RW1: ε_t is independent and identically distributed (iid) $(0, \sigma^2)$. Not realistic. (Old tests: Cowles and Jones (1937).

- RW2: ε_t is independent (allows for heteroskedasticity). Test using filter rules, technical analysis. (Alexander (1961, 1964), Fama (1965).

- RW3: ε_t is uncorrelated (allows for dependence in higher moments). Test using autocorrelations, variance ratios, long horizon regressions.

Autocorrelation Tests

- Assume r_t is covariance stationary and ergodic. Then

$$\gamma_k = cov(r_t, r_{t-k})$$

$$\rho_k = \gamma_k / \gamma_0 .$$

With sample estimates g_k and r_k.

- Under RW1 (and some assumptions)

$$E[r_k] = -(T-k)/(T(T-1))+O(T^2)$$

$$Cov[r_k, r_l] = (T-k)/T^2 + O(T^2) \qquad \text{(if } k=l \neq 0)$$

$$= O(T^2) \qquad\qquad \text{(otherwise)}$$

$$T^{1/2} r_k \rightarrow N(0,1)$$

<u>Note</u>: The sample correlation coefficients are negatively biased in finite samples. See Fuller (1976).

• Box-Pierce Q-statistic tests a joint hypothesis. The first m autocorrelations are equal to zero.

That is, $H_0: \rho_1 = \cdots = \rho_m = 0$.

The usual Q-statistic version is the Ljung-Box (1978) version. (It has better small sample properties.)

. Under RW1 and using the asymptotic distribution of r_k,
$$Q = T(T+2) \Sigma_k (r_k/(T-k)) \sim \chi^2_m$$

Variance Ratios

- Intuition. For all 3 RW hypotheses, the variance of RW increments is linear in the time interval. If the interval is twice as long, the variance must be twice as big. Under RW1, the 2-period variance ratio satisfies
  $$VR(2) = var(r_t(2))/(2\ var(r_t)) = var(r_t + r_{t-1})/(2\ var(rt)) = 2\sigma^2/2\sigma^2 = 1.$$
- If r_t is a covariance stationary process then
  $$VR(2) = (var(r_t) + var(r_{t-1}) + 2cov(r_t, r_{t-1}))/(2\ var(r_t))$$
  $$= (2\sigma^2 + 2\gamma_1)/2\sigma^2 = 1 + \rho_1$$

Three cases:
- $\rho_1 = 0 \Rightarrow VR(2) = 1$
- $\rho_1 > 0 \Rightarrow VR(2) > 1$ (mean aversion)
- $\rho_1 < 0 \Rightarrow VR(2) < 1$ (mean reversion)

• The intuition generalizes to the q−period case (under stationarity):
$$VR(q) = var(r_t(q))/(q \ var(r_t)) = 1 + 2 \Sigma_k (1-(k/q)) \rho_k$$
$$r_t(q) = r_t + r_{t-1} + \cdots + r_{t-q+1}$$
The VR(q) is a particular linear combination of the 1st (q-1) autocorrelation coefficients (with linearly declining weights).

<u>Note 1</u>: Under RW1, V R(q) = 1.

<u>Note 2</u>: For stationary and ergodic returns with a 1-summable Wold representation
$$r_t = \mu + \Sigma_{j=0} \psi_j \varepsilon_{t-j}, \qquad \varepsilon_t \sim iid(0, \sigma^2), \psi_0 = 1, \Sigma j |\psi j| < \infty$$

it can be shown that
$$\lim_{q\to\infty} VR(q) = \sigma^2 \psi(1)^2/\gamma_0 = lrv(r_t)/var(r_t)$$
$$= \text{long-run variance/short-run variance}$$

<u>Note 3</u>: Under RW2 and RW3, VR(q) = 1 provided
$$1/T \Sigma_t var(r_t) \to \bar{\sigma}^2 > 0$$

• **Lo and MacKinlay's Test Statistics**

Lo and MacKinlay (1988, 1989) present a number of test statistics for testing the RW hypothesis based on the estimated variance ratio:

$$\text{Est. } VR(q) = \text{Est. } var(r_t(q))/(q \ \text{Est. } var(r_t))$$

The form of the statistic depends on the particular random walk model (RW1, RW2 or RW3) assumed under the null hypothesis.

Check CLM for the details.

Other Tests

- Cross-autocorrelations and Lead-lag Relations: Lo and MacKinlay (1990).
- Long Range Dependence
  - ARFIMA models: Granger and Joyeux (1980) and Hoskings (1981).
  - Rescaled Range (R/S statistics): Mandelbrot (1972, 1975), Mandelbrot and Wallis (1969).
- Mean Reversion Tests: Changes in returns must be negatively serially correlated at some frequency for the mean correction to occur. predictable. Fama and French (1988).

Empirical Evidence: Autocorrelations and VR tests

- Tests based on CRSP value-weighted and equal weighted indices, individual securities from 1962-1994.
- Daily, weekly and monthly returns from VW and EW indices show significant (positive) autocorrelation.
- $VR(q) > 1$ statistics reject RW3 for EW index but not VW index. Market capitalization or size may be playing a role. Rejection of RW stronger for smaller firms. Their returns more serially correlated.
- For individual securities, $VR(q)<1$, suggesting small and negative correlations (and not significant).
- Q: How can portfolio $VR(q) > 1$ when individual security $VR(q) < 1$?